

The Period-Ratio and Mass-Ratio Correlation in Extra-Solar Multiple Planetary Systems

Ing-Guey Jiang^a, Li-Chin Yeh^b, Wen-Liang Hung^b

^a*Department of Physics and Institute of Astronomy,
National Tsing Hua University, Hsin-Chu, Taiwan*

^b*Department of Applied Mathematics,
National Hsinchu University of Education, Hsin-Chu, Taiwan*

ABSTRACT

Employing the data from orbital periods and masses of extra-solar planets in 166 multiple planetary systems, the period-ratio and mass-ratio of adjacent planet pairs are studied. The correlation between the period-ratio and mass-ratio is confirmed and found to have a correlation coefficient of 0.5303 with a 99% confidence interval (0.3807, 0.6528). A comparison with the distribution of synthetic samples from a Monte Carlo simulation reveals the imprint of planet-planet interactions on the formation of adjacent planet pairs in multiple planetary systems.

Key words: planetary systems, statistical method

1. Introduction

It is now about two decades after the first discovery of extra-solar planets. The rapid development of observational and theoretical research on planets has been remarkable. The rich properties of discovered extra-solar planetary systems provide many implications on the major processes of planetary formation. It is indeed exciting that the detections of many extra-solar planetary systems make it possible to understand how our solar system was originally formed.

Because the formation and evolution of planets are closely related to their orbital periods and masses, the distribution of extra-solar planets (exoplanets) on the period-mass (or semimajor-axis versus mass) plane has become one of the most important diagrams to be investigated. For example, Zucker & Mazeh (2002) proposed a possible correlation between planet masses and their orbital periods. Jiang et al. (2006) used the clustering analysis of exoplanets on the period-mass plane and found that the locations of cluster centers are

consistent with the period-mass correlation. After Tabachnik & Tremaine (2002) presented period and mass distributions and the implied fractions of stars with exoplanets, Jiang et al. (2007, 2009) further studied the coupled period-mass functions and the period-mass correlation simultaneously for the first time in this field. They employed the copula modeling method (Scherrer et al. 2010, Takeuchi 2010) and obtained period-mass functions successfully. They also showed that there was a positive correlation between mass and period. Finally, the selection effect was considered, and the fundamental period-mass functions were constructed by Jiang et al. (2010).

Theoretically, the planet population synthesis performed by Mordasini et al. (2009) showed that gas giant planets are likely to have larger orbital periods than terrestrial planets. Neglecting planet-planet interactions, the results reported by Mordasini et al. (2009) led to the period-mass correlation naturally, and thus, give a physical explanation for the observational period-mass correlation with samples from both detected single and multiple planetary systems.

On the other hand, focusing on the interaction between adjacent planets in multiple systems, a planetesimal accretion model with a concept of angular momentum deficit was proposed by Laskar (2000). This theory has a prediction on the period-ratio and mass-ratio relation between two consecutive planets. That is, for an initial disk of planetesimals with a mass density distribution $\rho(a) = ca^s$, where a is the semi-major axis of planetesimals, c is a constant, and s is the power index, the mass ratio of consecutive planets can be expressed as:

$$\frac{m_o}{m_i} = \left[\frac{p_o}{p_i}\right]^{(2s+3)/9}, \quad (1)$$

where m_o and m_i are the mass of the outer and inner planets and p_o and p_i are the periods of the outer and inner planets. Mazeh and Zucker (2003) presented a first attack on this relationship between the period ratio and mass ratio of adjacent planets. They found a surprisingly tight correlation between period ratio and mass ratio. (For convenience, this is called the PRMR correlation hereafter.) The Pearson’s correlation coefficient between the logarithms of period ratio and mass ratio was 0.9415. However, there were only about ten samples in their study. Due to that, the correlation coefficient was greater than 0.9. It would be very interesting to re-examine this possible correlation with much more currently available samples.

Moreover, it can be easily shown that when the period-mass correlation follows a pure power-law, it will lead to a PRMR correlation with exactly the same law. However, in a case where the PRMR correlation is much tighter than the period-mass correlation, the PRMR correlation shall be driven by an additional mechanism. This is another main reason that we investigate this problem of possible PRMR correlation here.

2. The Method

According to the Extrasolar Planets Encyclopedia (<http://exoplanet.eu/catalog-all.php>), on 9th Oct. 2014, there were more than four hundred detected multiple planetary systems. However, among these, a huge number of multiple planetary systems, detected by the Kepler Project, did not have information on the mass of any individual planet. Note that “mass” means the value of projected mass in this letter. After removing these systems and a system with one unknown planetary orbital period, there were 166 multiple planetary systems left for our study.

As in Mazeh and Zucker (2003), the logarithms of the period ratio and mass ratio of adjacent planets in extra-solar multiple planetary systems are employed as two independent variables (x, y) here. That is, pr is the outer-to-inner period ratio, mr is the outer-to-inner mass ratio, and we define $x = \ln(pr)$ and $y = \ln(mr)$.

When the system only consists of two planets, we calculate the period ratio and mass ratio between the outermost and the innermost one; when the multiple system consists of three planets, we compute two sets of ratios: one set of ratios between the intermediate planet and the innermost one, and another set of ratios between outermost and the intermediate one. When the multiple system consists of L ($L > 3$) planets, we obtain $(L - 1)$ sets of the period ratio and mass ratio in the same way. Note that some of the above 166 multiple planetary systems contain one or two planets with unknown masses, and thus, those pairs related to them cannot be used. We get 236 useful pairs of the period ratio and mass ratio from our data of 166 multiple planetary systems.

In order to check whether there are any pairs with period ratios associated with resonances, the histogram of period ratio pr is shown in Fig. 1(a). The maximum pr is more than one thousand. To make the main part of the plot clear, Fig. 1(a) is plotted up to $pr = 10$ only. This justifies why the natural logarithms of the period ratio and mass ratio are used as variables. With the bin-size being 0.01, there are only two places that have numbers of samples greater than three. One is around $pr = 1.5$ (3:2 resonance) and another is around $pr = 2$ (2:1 resonance). Note that the small peak, located between the above two, which has three samples has nothing to do with any first order resonance. The number of samples is smaller than three for all the rest up to the maximum pr .

Figs. 1(b)-(c) are the plots near $pr = 1.5$ and $pr = 2$, respectively. Since the samples in the connected bumps from $pr = 1.45$ to $pr = 1.53$ and from $pr = 1.99$ to $pr = 2.09$ could be associated with 3:2 or 2:1 resonances, they are excluded from our sample set. Finally, we end up with 188 pairs of the period ratio and mass ratio here.

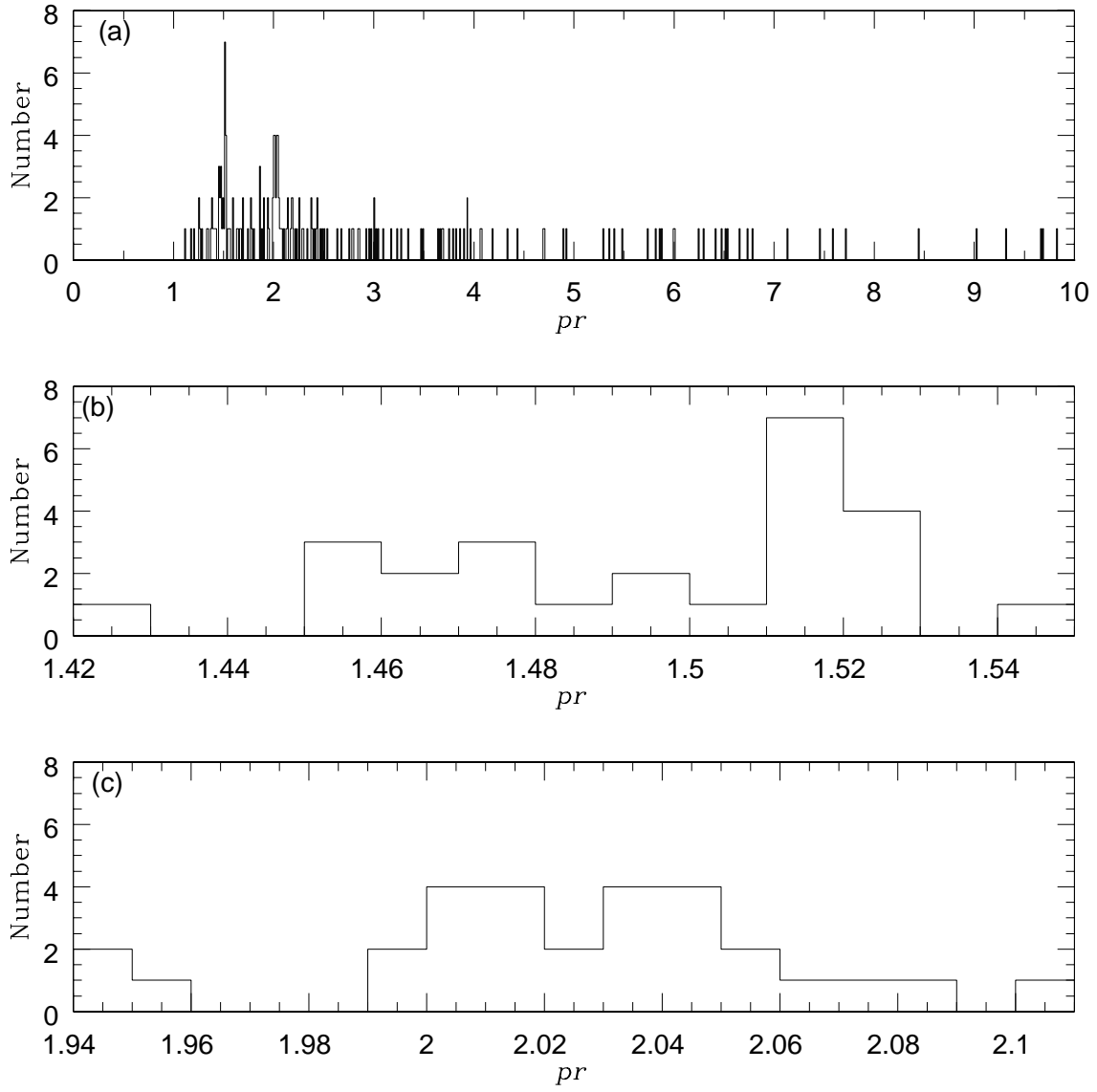


Fig. 1.— (a) histogram of pr up to $pr = 10$, (b) histogram of pr near 3:2 resonance, (c) histogram of pr near 2:1 resonance.

3. Results

Fig. 2(a) shows the locations of 188 samples on the $x - y$ plane and presents a positive correlation. The straight line is a best least-square linear fitting, which is $y = 0.536x - 0.398$. From the formula of Pearson’s correlation coefficient, we obtain the correlation coefficient as 0.5303. Moreover, through the Fisher’s z-transformation, the 99% confidence interval of the correlation coefficient is determined to be (0.3807, 0.6529).

In order to further test the statistical significance of this possible correlation, a Monte Carlo simulation of independent sampling from x and y is performed. That is, from Fig. 2(a), one x value is randomly picked and one y value is also picked independently. They form a new (x, y) pair which might or might not be one of the original real data pairs. After we get 188 synthetic pairs, the correlation coefficients of these samples are calculated. This process is repeated for 10^6 times, and it is found that none of the 10^6 correlation coefficients is larger than 0.5303. This reconfirms the positive PRMR correlation.

Moreover, as we mentioned in §1 a period-mass correlation could lead to a PRMR correlation. In order to test whether the PRMR correlation we found here is purely derived from the period-mass correlation or not, we perform another Monte Carlo simulation to produce synthetic sets of samples of period-ratio and mass-ratio from all planets in our employed 166 multiple planetary systems. That is, we create 188 sample pairs through a random process, and the pairs are not necessarily adjacent planets. The period-ratio and mass-ratio derived from these samples will give a result which contains a pure period-mass correlation but no information about planet-planet interactions of adjacent planets. Out of 10^6 random synthetic sets, only 25255, i.e. about 2.5%, yield correlation coefficients larger than 0.5303. The average value of these 10^6 correlation coefficients is 0.4241.

Using one synthetic set with the correlation coefficient being 0.4241 as an example, Fig. 2(b) shows the locations of 188 synthetic samples on the $x - y$ plane, and the dotted line is a best least-square linear fitting. To estimate the level of scattering, the root-mean-square distance between the points and the line is calculated at 2.426. This is much larger than the corresponding value, 1.249, from Fig.2(a). The smaller scattering and the larger correlation coefficient for the result of adjacent pairs confirm the imprint of planet-planet interactions on our observational PRMR correlation.

4. Conclusions

Using the data from 166 extra-solar multiple planetary systems, the correlation between the period-ratio and mass-ratio of adjacent planets is confirmed. The correlation coefficient

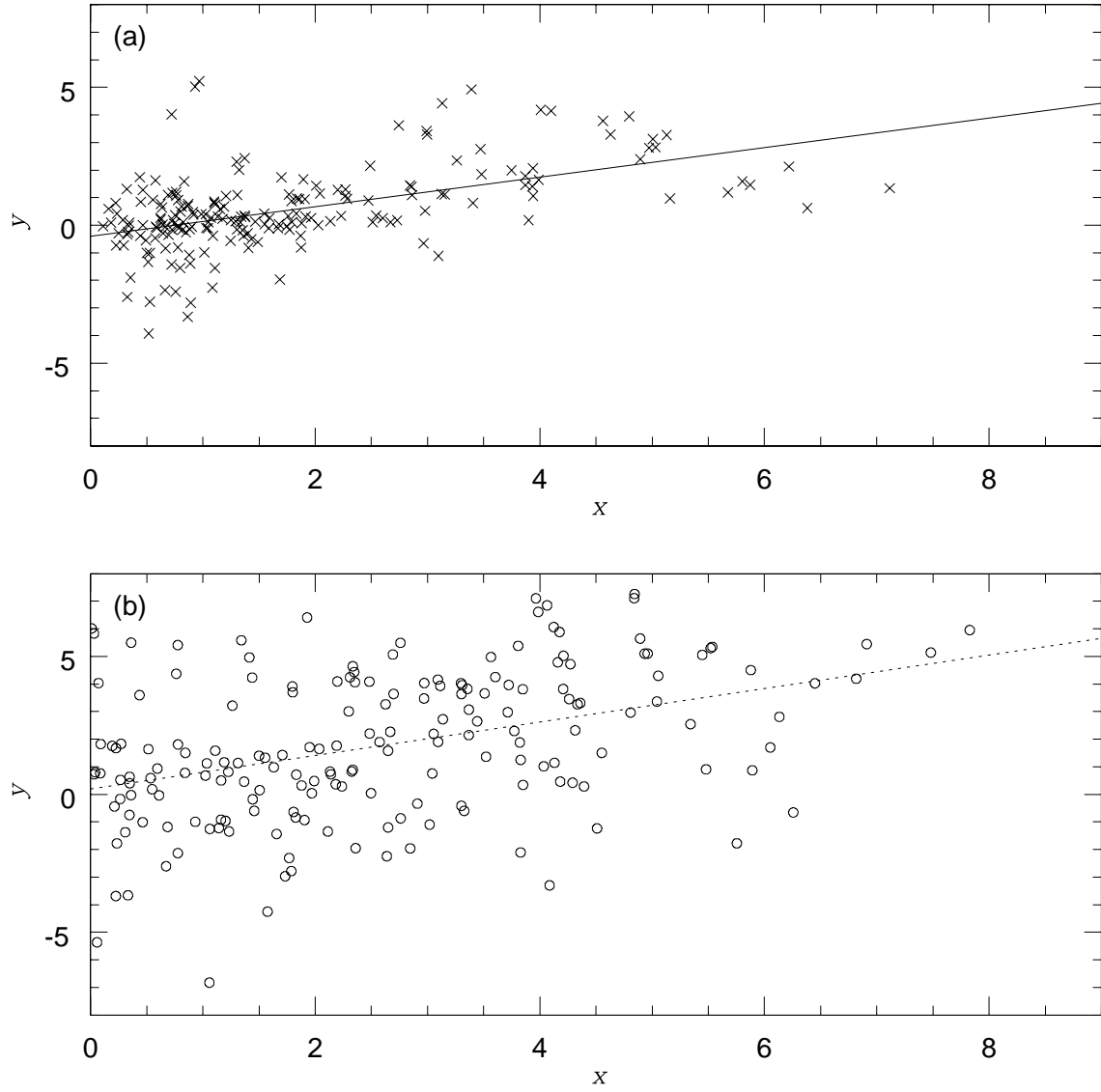


Fig. 2.— (a) The distribution of 188 observational samples on $x - y$ plane, (b) The distribution of one set of 188 synthetic samples on $x - y$ plane. The points are the locations of samples and the lines are the best least-square linear fitting.

is 0.5303 with a 99% confidence interval (0.3807, 0.6528).

How can we understand this PRMR correlation? The core-accretion model of planetary formation has been improved after considering the orbital migration and is likely to be a standard theory to describe the planetary formation (Alibert et al. (2009)). Based on that model, Mordasini et al. (2009) performed planet population synthesis for typical planetary systems and showed the planet formation tracks in their Fig. 8, which can be summarized as: (1) planets initially formed at less than 5 AU from the central star will grow to about $10 M_{\oplus}$; (2) planets initially formed well beyond 5 AU from the central star are likely to become gas giant planets; (3) those initially formed between 3 and 7 AU can migrate inward significantly and be within 1 AU of the central star; and (4) those initially formed beyond 7 AU would migrate inward but would not be within 1 AU of the central star.

The above results imply that gas giant planets are likely to have larger orbital periods than terrestrial planets in planetary systems. Thus, this leads to the period-mass correlation as studied in Jiang et al. (2009). However, because the planet-planet interactions are not considered in Mordasini et al. (2009), the above theory can only be responsible for the PRMR correlation of synthetic samples in Fig. 2(b).

The theory proposed in Laskar (2000), which addresses the period ratio and mass ratio between consecutive planets in multiple planetary systems, explains why the PRMR correlation in Fig. 2(a) is tighter than the synthetic one in Fig. 2(b). For an initial disk of planetesimals with mass density distribution $\rho(a) = ca^s$, Laskar (2000) only considered $s = 0, -0.5, -1$, and -1.5 . Our linear fitting line in Fig. 2(a) is closer to, but not the same as, the result with uniform density distribution of planetesimals, i.e. $s = 0$. The discrepancies are expected because the initial disk of planetesimals in different multiple planetary systems might not have the same mass density distribution. A complete theory which considers these diversities of multiple planetary systems and includes both the planet-planet interactions and those planet formation tracks in Mordasini et al. (2009) should be able to produce the observational period-ratio and mass-ratio distribution successfully. Nevertheless, the imprint of planet-planet interactions on the formation of consecutive planet pairs in multiple planetary systems has been revealed through the analysis here.

Acknowledgments

We are thankful for the referee’s suggestions. This work is supported in part by the Ministry of Science and Technology, Taiwan.

REFERENCES

- Alibert, Y., Baraffe, I., Benz, W., Laughlin, G., Udry, S. 2009, *Structure Formation in Astrophysics*, Edited by Gilles Chabrier, page 378, Cambridge University Press, Cambridge
- Jiang, I.-G., Yeh, L.-C., Hung, W.-L., Yang, M.-S. 2006, *MNRAS*, 370, 1379
- Jiang, I.-G., Yeh, L.-C., Chang, Y.-C., Hung, W.-L. 2007, *AJ*, 134, 2061
- Jiang, I.-G., Yeh, L.-C., Chang, Y.-C., Hung, W.-L. 2009, *AJ*, 137, 329
- Jiang, I.-G., Yeh, L.-C., Chang, Y.-C., Hung, W.-L. 2010, *ApJS*, 186, 48
- Laskar, J. 2000, *Physical Review Letters*, 84, 3240
- Mazeh, T., Zucker, S. 2003, *ApJ*, 590, L115
- Mordasini, C., Alibert, Y., Benz, W. 2009, *A&A*, 501, 1139
- Scherrer, R. J., Berlind, A. A., Mao, Q., McBride, C. K. 2010, *ApJ*, 708, L9
- Tabachnik, S., Tremaine, S. 2002, *MNRAS*, 335, 151
- Takeuchi, T. T. 2010, *MNRAS*, 406, 1830
- Zucker, S., Mazeh, T. 2002, *ApJ*, 568, L113